$\xi(t)$ - Gaussian white noise, $\langle \xi(t)\xi(t') \rangle = \delta(t-t')$.

The random process described by the Stochastic Differential Equation (SDE)

$$\frac{dx}{dt} = a(x,t) + b(x,t)\xi(t) \tag{1}$$

is equivalent to the Fokker-Planck equation for the probability distribution function $p(x,t)$

$$\frac{\partial}{\partial t} p(x,t) = -\frac{\partial}{\partial x}\left[a(x,t)p(x,t)\right] + \frac{1}{2}\frac{\partial^2}{\partial x^2}\left[b(x,t)^2 p(x,t)\right] \tag{2}$$

Handbook of Stochastic Methods, Gardiner, Chap. 4

The pitch angle scattering operator

$$\begin{aligned}\frac{\partial f}{\partial t} &= \nu_d \frac{\partial}{\partial \lambda}(1-\lambda^2)\frac{\partial f}{\partial \lambda} \\ &= \frac{\partial}{\partial \lambda}\left[2\nu_d \lambda f\right] + \frac{1}{2}\frac{\partial^2}{\partial \lambda^2}\left[2\nu_d (1-\lambda^2) f\right]\end{aligned} \tag{3}$$

corresponds the Langevin equation

$$\frac{d\lambda}{dt} = -2\nu_d \lambda + \left[2\nu_d(1-\lambda^2)\right]^{1/2}\xi(t) \tag{4}$$

The white noise $\xi(t)$ can be implemented as a random change in λ

$$\triangle\lambda = \left[2\nu_d(1-\lambda^2)\right]^{1/2} N(0,1) \tag{5}$$

$N(0,1)$ is a Gaussian random number with mean 0 and variance 1.

A more simple way

$$\lambda_{\text{new}} = \lambda_{\text{old}}(1-2\nu_d\triangle t) \pm \left[2(1-\lambda_{\text{old}}^2)\nu_d\triangle t\right]^{1/2} \tag{6}$$

with equal probablity for plus and minus. *Boozer and Kuo-Petravic, 1981.*

Because $p_\parallel = v_\parallel - \frac{e}{m}A_\parallel$ is used, an additional term appears in the weight equation due to $L(f_0(p_\parallel))$.

$$\begin{aligned} f_0(p_\parallel) &= \frac{n_0}{(2\pi)^{3/2}v_T^3}e^{-\frac{1}{2}m(v_\perp^2+p_\parallel^2)/T} \\ &\approx \frac{n_0}{(2\pi)^{3/2}v_T^3}e^{-\frac{1}{2}m(v_\perp^2+v_\parallel^2)/T} + \frac{ev_\parallel A_\parallel}{T}f_M \end{aligned} \tag{7}$$

$L[f_M(v)] = 0$,

$$\begin{aligned} L(v_\parallel) &= \nu_d\frac{\partial}{\partial\lambda}(1-\lambda^2)\frac{\partial}{\partial\lambda}(v\lambda) \\ &= -2\nu_d v_\parallel \end{aligned} \tag{8}$$

$$\begin{aligned} L(f_e) &= L(f_0(p_\parallel)) + L(\delta f) \\ &= -2\nu_d e\frac{v_\parallel A_\parallel}{T}f_M + L(\delta f) \end{aligned} \tag{9}$$

In the code the collision operator is given by

$$C_L(f_e) = \nu_e\frac{1}{2}\frac{\partial}{\partial\lambda}(1-\lambda^2)\frac{\partial}{\partial\lambda}f_e \tag{10}$$

$$\nu_e = \frac{n_{0\mathrm{e}}e^4\ln\Lambda}{4\pi\epsilon_0^2 m_e^2 v^3}\left(Z_{\mathrm{eff}} + H_{\mathrm{ee}}\left(\sqrt{m_e v^2/2T_{0\mathrm{e}}}\right)\right) \tag{11}$$

with $H_{\mathrm{ee}}(x) = \dfrac{e^{-x^2}}{\sqrt{\pi}x} + (1 - \dfrac{1}{2x^2})\mathrm{erf}(x)$.

$\nu_d = \nu_e/2$

$$\nu_{\mathrm{ei}} \equiv \frac{n_{0\mathrm{e}} e^4 \ln \Lambda}{4\pi \epsilon_0^2 m_e^2 v_{\mathrm{Te}}^3} \tag{12}$$

$$\begin{aligned} \nu_e &= \nu_{\mathrm{ei}} \frac{v^3}{v_{\mathrm{Te}}^3} \left(Z_{\mathrm{eff}} + H_{\mathrm{ee}} \left(\sqrt{m_e v^2 / 2T_{0\mathrm{e}}} \right) \right) \\ &= \nu_{\mathrm{ei}} \left(\frac{T_e}{\epsilon} \right)^{1.5} \frac{1}{2^{1.5}} \left(Z_{\mathrm{eff}} + H_{\mathrm{ee}} \left(\sqrt{m_e v^2 / 2T_{0\mathrm{e}}} \right) \right) \end{aligned} \tag{13}$$